QUANTITATIVE REASONING BY UNDERGRADUATE STUDENTS AT THE UNIVERSITY OF VIRGINIA: 2013-14 ASSESSMENT

Executive Summary

The State Council of Higher Education for Virginia (SCHEV) requires that all institutions of higher education assess undergraduate core competencies. Each year, the University assesses one of six designated competencies.

In 2013-2014, the University's Office of Institutional Assessment and Studies initiated planning for an institution-wide assessment of undergraduate competency in quantitative reasoning. The University Undergraduate Quantitative Reasoning Competency Assessment Committee was convened to provide oversight. The final report will be sent to the Provost and deans and a summary report to SCHEV.

The assessment employed the same instrument that had been designed for the previous assessment in 2007-2008. Designed to assess fourth-year students' competence in three areas—general mathematics, statistical reasoning, and calculus/trigonometry, the test was composed of 30 questions. A representative sample of 244 fourth-year students from five undergraduate schools and 179 first-year students consented to take the test.

Graduating fourth-years met the standards for competency set by the committee. Twenty-four percent of fourth-years were classified as highly competent, 81% classified as competent or above, and 96% as minimally competent or above. The performance of fourth-year engineering/science/math majors was particularly high with 35% of engineering/science/math majors achieving high competence, 86% achieving competence or above, and 90% achieving minimal competence or above.

First-year students' performance was used to assess value-added; the mean difference in performance between first-years' overall performance and fourth-years' overall performance was not significant. However, fourth-year engineering/science/math majors outperformed first-years overall. Fourth-year students did outperform first-year students on statistics/statistical reasoning questions, and first-year students *almost* outperformed fourth-year students on calculus/trigonometry questions.

These results, both for competency assessment and value-added, are highly similar to those from the previous (2007-2008) assessment.

Findings

1. Our fourth-year students, on the whole, are competent in quantitative reasoning.
2. Evidence of value-added is not present, except with regard to statistics/statistical reasoning, and engineering/science/math majors.
3. Fourth-year students' performance in 2014 did not differ substantially from that of fourth-year students in 2008.

Background

In 2004, the first assessment of quantitative reasoning employed a national standardized test[1] that provided an overall comparison of U.Va. results with those from other institutions. Such a "broad brush" approach, however, provided no useful information other than comparative overall competency. Subsequently, in 2007-2008, a faculty committee charged with overseeing the second university-wide assessment of undergraduate quantitative reasoning determined to create an assessment process that would yield a deeper understanding of students' competency. This committee, representing major disciplines and each undergraduate school, developed a definition of quantitative reasoning, defined associated learning outcomes and standards for performance, and created a 30-item test that could be administered to students in a single sitting.

The test included 30 multiple-choice test questions (Appendix A) each of which mapped onto at least one of the learning outcomes (Appendix B) and onto one of three major topics: general mathematics/mathematical reasoning, statistics/statistical reasoning, and calculus/trigonometry (Appendix C). Each question was ranked according to difficulty: easy, moderate, or hard. One outcome—communicate mathematical information symbolically, visually, numerically, and verbally—was not represented on the test.

Administered to first- and fourth-year students, the test served well in identifying areas of relative strength and weakness in quantitative reasoning and topics for which value-added was evident.

For the 2013-14 assessment, the test was used again to obtain a second "read" on its usefulness for understanding students' ability to employ quantitative reasoning. The definition, learning outcomes, and standards set by the 2007-8 committee were maintained without change.

Definition, Outcomes, Standards

Definition

Quantitative reasoning is correctly using numbers and symbols, studying measurement, properties, and the relationships of quantities, or formally reasoning within abstract systems of thought to make decisions, judgments, and predictions.

Student Learning Outcomes

A graduating fourth-year undergraduate at the University of Virginia will be able to:

1. Interpret mathematical models such as formulas, graphs, tables, and schematics, and draw inferences from them.
2. Communicate mathematical information symbolically, visually, numerically, and verbally.
3. Use arithmetical, algebraic, geometric, and analytic methods to solve problems.
4. Estimate and check answers to mathematical problems in order to determine reasonableness.
5. Solve word problems using quantitative techniques and interpret the results.

[1] ACT CAAP: Collegiate Assessment of Academic Proficiency

6. Apply mathematical/statistical techniques and logical reasoning to produce predictions, identify optima, and make inferences based on a given set of data or quantitative information.
7. Judge the soundness and accuracy of conclusions derived from quantitative information, recognizing that mathematical and statistical methods have limits and discriminating between association and causation.
8. Solve multi-step problems.
9. Apply statistics to evaluate claims and current literature.
10. Demonstrate an understanding of the fundamental issues of statistical inference, including measurement and sampling.

Standards/Level of Performance Expected

The following standards have been established for graduating fourth-years:

- 25% of fourth-years are expected to be highly competent (score ≥23 out of 30);
- 75% competent or above (score 16-22);
- 90% minimally competent or above (score 11-15).

Methods

Instrument

A faculty committee representing major disciplines and each undergraduate school had developed the single-setting instrument to assess quantitative reasoning. The instrument, comprised of 30 multiple-choice questions, was administered at scheduled one-hour test sessions in February 2014.

The test questions were designed to address three major topics in quantitative reasoning (Appendix C):

1. general mathematics/mathematical reasoning (including arithmetic, geometry, and algebra[2]) (16 questions);
2. statistics/statistical reasoning (9 questions);
3. calculus/trigonometry (5 questions).

Sampling

First- and fourth-year students from six undergraduate schools at the University (Architecture, BIS/SCPS, Commerce, Engineering, Nursing, and the College of Arts and Sciences) were invited to take the assessment. To reduce the chance of self-selection, invitations did not identify the topic of the assessment. Two hundred and forty-four (244) fourth-year students and 300 first-year students participated in the assessment. No students from the BIS program of the School of Continuing and Professional Studies (SCPS) participated in the assessment, so the resulting fourth-year sample is underrepresented by SCPS. To improve representativeness of the sample of fourth-years, 29 fourth-

[2] The reliability of the general math/math reasoning category was acceptable (KR-20 = .76) but the reliabilities of the calculus/trig (KR-20=.45) and statistics categories were low (KR-20 = .39).

years were randomly removed from the over-represented schools of Engineering, Commerce, and Nursing, yielding a final sample of 215 fourth-year students.

Fourth-Year Representative Sample

	Count	Percent
Architecture	6	2.7%
Arts & Sciences Hum/FA	26	12.1%
Arts & Sciences Science	56	26.0%
Arts & Sciences Social Science	57	26.3%
Commerce	18	8.6%
Education	4	1.8%
Engineering	38	17.9%
Leadership & Public Policy	2	1.0%
Nursing	7	3.2%
Total	215	100.0%

Confidentiality and Compensation

Only students who consented to volunteer were assessed. Confidentiality was ensured. Students who participated were given a $20 gift certificate to Amazon.com once they completed the test.

Results

Competency Results – Fourth Years

Overall, fourth-year students performed well on the quantitative reasoning assessment:

- Nearly all (96%) fourth-years achieved scores reflecting minimal competence or above (score ≥11), exceeding the standard for minimal competency.
- Also exceeding the standard, 81% of fourth-year students answered half or more of questions correctly, thereby scoring as competent.
- Twenty-four percent of students essentially met the standard for high competence (score ≥23), scoring one percentage point lower than the standard.
- Engineering/science/math majors substantially exceeded all three standards.

Table 1 Quantitative Reasoning Competency of Graduating Fourth-Years

	Minimally competent (score 11+) (answering 1/3 or more questions correctly)	**Competent (score 16+)** (answering 1/2 or more questions correctly)	**Highly competent (score 23+)** (answering 3/4 or more questions correctly)
Standards for competency	90%	75%	25%
4th Years (all)	96%	81%	24%
4th Years (Engineering, Science, Math)	99%	86%	35%

Comparison: First-Year vs. Fourth Year

Three hundred (300) first-year students participated in the assessment. Among those from the College of Arts & Sciences, approximately half were asked their *intended* major and were subsequently coded as *intending to major in* science, social science, humanities/fine arts, commerce or leadership/policy. First-year students who intended to major in science were overrepresented: twice as many intended science majors participated in the assessment as would have been predicted based on the population of fourth-year declared majors. To create a representative sample of first-years to compare to fourth-years, 92 students were randomly removed to yield a representative sample of 208 first-year students.

Table 2: Mean Total Scores: First-year vs. Fourth-year Students

	First-years		**Fourth-years**	
N	208		215	
	Score	Standardized*	Score	Standardized*
Mean total score	18.5	62	19.3	64
Standard deviation	4.5	15	4.4	15
Median total score	19.0	63	20.0	67
Top 25% cut-off score	22.0	73	22.0	73
50% cut-off score	19.0	63	20.0	67
Bottom 25% cut-off score	16.0	53	16.0	53

*Note: scores standardized to 100-point scale

Mean differences in overall test performance between first-years and fourth-years were not statistically significant. It should be noted though that fourth-year engineering/science/math majors[3] (*M*=20.12) significantly outperformed first-years (*M*=18.36)[4]. This result continued to be significant after controlling for SAT Math score.

Overall, fourth-years outperformed first-years on three questions, a general math/math reasoning question and two statistics/statistical reasoning questions. First-years, however, outperformed fourth-years on three general math/math reasoning and on three calculus/trigonometry questions. First-year students are likely to have taken courses recently, including Advanced Placement courses, in algebra, calculus, and/or trigonometry, but not in probability/statistics[5]; indeed, nearly all incoming U.Va. survey respondents in 2013 had taken AP algebra II and pre-calculus/trigonometry; 64-75

[3] The mean score for engineering majors was 21.76, the mean score for science/math majors was 19.08.

[4] $F(2, 504)=6.48$, $p<.01$, partial eta-squared=.03, power=.91

[5] CIRP survey: https://avillage.web.virginia.edu/iaas/survey/data/2013-14/TFS_2013_PDF_PROFILE.pdf

percent had taken calculus or AP calculus, whereas only one-third had taken probability/statistics or AP probability/statistics.

Mean differences between fourth-year and first-year performance on the general math/math reasoning and calculus/trigonometry question categories were not significant, although mean differences in performance on the calculus/trig questions *approached* significance, with first-years almost outperforming fourth-years. On the statistics/statistical reasoning questions, fourth-years outperformed first-years ($p<.001$). Fourth-year students are likely to have had greater exposure to, and thus greater competence in statistics and statistical reasoning because these concepts are more often covered in a variety of disciplines and college courses than those in high school. Table 3 presents standardized total scores for each topic of questions by student academic year.

Table 3: Mean Standardized Total Scores by Topic: First-year vs. Fourth-year Students

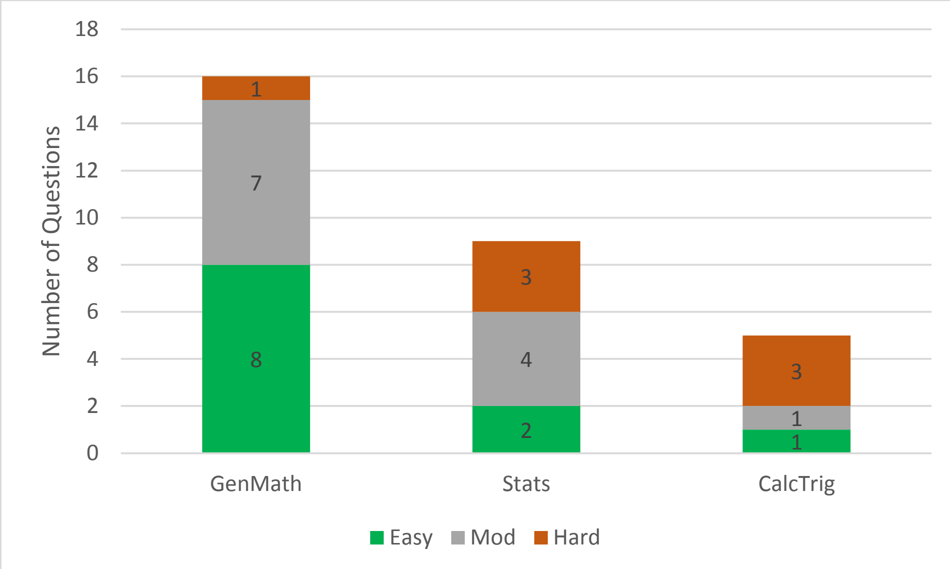
	General Mathematics:		**Statistics/Statistical Reasoning:**		**Calculus/ Trigonometry:**	
	Mean Score*	S.D.	Mean Score*[6]	S.D.	Mean Score*	S.D.
First Years (n=208)	69	18	56	18	48	23
Fourth Years (n=215)	71	17	65	18	42	25

Note: scores standardized to 100-point scale

Analysis of results by topic is complicated by differences in difficulty among the question sets, however. For example, of the 16 General Math questions, half (8) are described as "easy," nearly half (7) as "moderate," and only one as "hard." In contrast, of the nine Statistics questions, about one-fifth (2) are "easy," nearly half (4) are "moderate," and one-third (3) are "hard" (Figure 1).

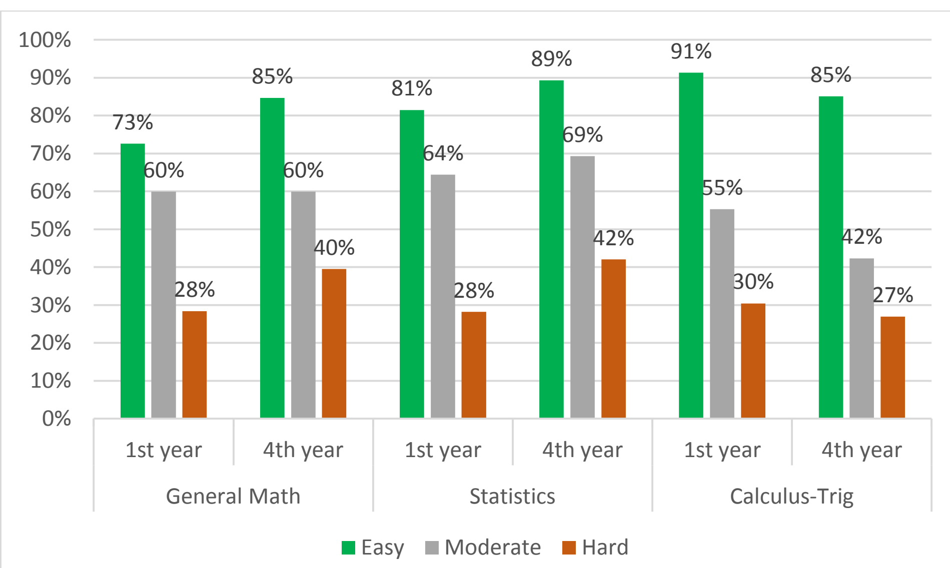
[6] $F(1, 505)=14.44$, $p<.001$, partial eta-squared=.03, power=.97

Figure 1: Composition of Test Topics by Number and Difficulty of Questions



Analysis by question difficulty within topic helps to illuminate in what ways fourth-year students outperformed first-years in Statistics and first-years nearly outperformed fourth-years in Calculus-Trigonometry (Figure 2). Remarkably, among the Statistics questions, the largest difference (14 percentage points) between first- and fourth-year students was for the three "hard" questions. Differences for the two "easy" (8 percentage points) and 4 "moderate" (5 percentage points) questions all favored fourth-years. In contrast, while first- and fourth-year performance could not be described as different on the three "hard" Calculus-Trigonometry questions (3 percentage points), greater differences were seen on two questions: the single "moderate" (13 percentage points) and the single "hard" question (6 percentage points). Caution in interpretation is warranted given the small number of Calculus –Trigonometry questions.

Figure 2: Percent Correct Answers by Difficulty within Topic, First-years vs. Fourth-years



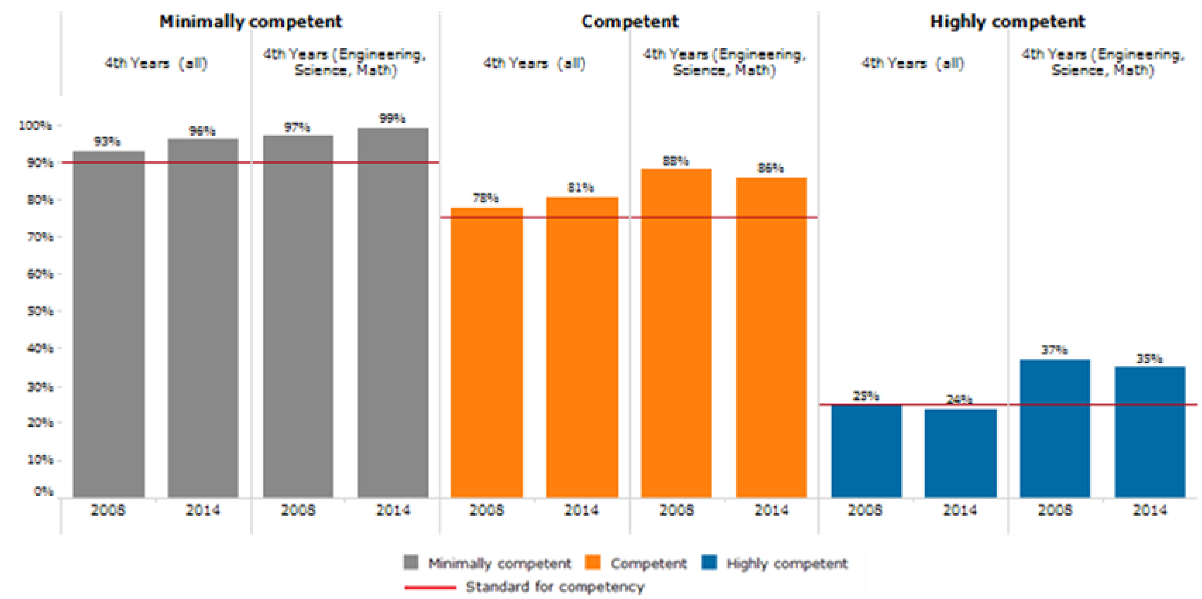
Results: 2008 vs. 2014

Just as fourth-year students met standards for competency in 2014, fourth-year students tested in 2008 did so as well. The same pattern emerged with math/science/engineering students in 2008 outperforming other fourth-year students (Table 4 and Figure 3).

Table 4: Quantitative Reasoning Competency of Graduating Fourth-Years: 2008 vs. 2014

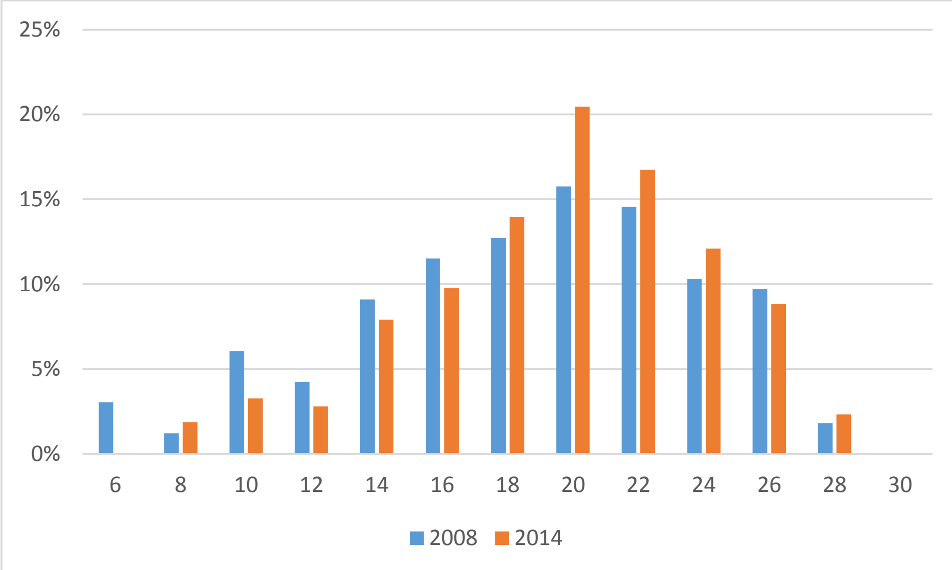
		Minimally competent (score 11+) (answering 1/3 or more questions correctly)	**Competent** (score 16+) (answering ½ or more questions correctly)	**Highly competent** (score 23-30) (answering ¾ or more questions correctly)
Standards for competency		90%	75%	25%
4th Years (all)	2014	96%	81%	24%
	2008	93%	78%	25%
4th Years (Engineering, Science, Math)	2014	99%	86%	35%
	2008	97%	88%	37%

Figure 3: Quantitative Reasoning Competency of Graduating Fourth-Years: 2008 vs. 2014



While mean scores between 2008 (mean=18.15, S.D.=5.18) and 2014 (mean=19.2, S.D.=4.49)) differed significantly *(p<.05),* the difference was not substantive (effect size small). The distributions are highly similar (Figure 4).

Figure 4: Comparison of Frequency Distributions of Total Scores: 2008 vs. 2014



Findings

1. U.Va. fourth-year students, on the whole, meet the standards for competency in quantitative reasoning.
2. Evidence of value-added is not present, except with regard to statistics/statistical reasoning, and by engineering/science/math majors.
3. Fourth-year students' performance in 2014 did not differ substantially from that of fourth-year students in 2008 overall.
4. The test appears to reliably assess students' proficiency in quantitative reasoning overall although caution is warranted when interpreting results by topic.

Appendices

Appendix A
Quantitative Reasoning Assessment Instrument

UNIVERSITY of VIRGINIA INSTITUTIONAL ASSESSMENT and STUDIES

Instructions:

Please do not begin until the IAS staff person has given you instructions to do so.

This assessment is for the purpose of improving undergraduate education and will never be a part of your academic record. Your individual score will never be reported but will be used to compute composite scores for the University, your school, and perhaps your major, if you have one.

The assessment is designed to measure your reasoning ability using quantitative information. While you may feel quantitative reasoning is not one of your particular strengths, we still want you to do your best and attempt to answer as many questions as possible. Many aspects of quantitative reasoning are covered, and some questions are more challenging than others. A wide range of students are being assessed (including first- and fourth-years). We do not expect that all students will be able to answer all questions correctly in the time allotted.

The assessment contains 30 multiple-choice questions. Again, please do your best and try to answer as many questions correctly as you can. You may use the calculator provided and you may write on this booklet.

Bubble Sheet:

On the bubble sheet provided, please fill in your **respondent key** where it says "UVA Computing ID" on the right. Write in the letters/numbers and also fill in the corresponding circles below. If you did not bring your respondent key with you, ask the IAS staff person to provide it to you. You do not need to put your name or any other information on the bubble sheet.

Completely fill in the circles corresponding to your answer for each of the 30 multiple-choice questions.

The following table provides the results of a census of a small metropolitan area for persons in households, their city or suburban residence, and whether they live in a household alone or with others. The data for those in the population who are seniors (those 65 years of age and older) is highlighted in the second table. Please refer to these tables to answer questions 1 and 2.

PERSONS IN HOUSEHOLDS	CITY	SUBURBAN	TOTAL
All persons:			
Living Alone	5,000	5,000	10,000
Not Living Alone	35,000	55,000	90,000
Total:	40,000	60,000	100,000
Seniors (65+) only			
Living Alone	2,000	3,000	5,000
Not Living Alone	4,000	7,000	11,000
Total:	6,000	10,000	16,000

The following questions seek specific probabilities for a person selected randomly from this metropolitan population.

(1) What is the probability that the person lives in the City?

A. 40%

B. 50%

C. 60%

D. 62.5%

E. 66%

(2) What is the probability that a person living alone is not 65 years of age or older?

A. 40%

B. 50%

C. 60%

D. 62.5%

E. 66%

(3) A store marks up the wholesale price of a shirt by 80%. After two months, the retail price of the shirt is reduced by 40%. What percentage above the wholesale price is the current shirt price?

A. 8%

B. 16%

C. 20%

D. 40%

E. 50%

(4) A business meeting includes 6 vice presidents. Each upon entering the room shakes hands with the others present. How many hand shakes are there?

A. 6

B. 10

C. 15

D. 36

E. 120

(5) There are 300 people waiting in line to buy concert tickets. Of these:

- 180 are male;
- 60 are female and not wearing shorts;
- 60% of the males are wearing shorts.

What is the total number of people in line wearing shorts?

A. 108

B. 120

C. 168

D. 180

E. 200

(6) If the average (mean) of three numbers *x*, *y*, and *z* II 12, then what is the average of *x*, *y*, *z*, and *I* = 9?

A. 10

B. 10.75

C. 11

D 11.25

E. 11.5

(7) The number of web sites on the Internet grew exponentially in the early 1990's, with the number of sites (in millions) given below:

Year	1990	1991	1992
Sites	2	6	18

Assuming that this rate of growth continued, how many sites were there in 1994?

A. 42 million

B. 54 million

C. 72 million

D. 102 million

E. 162 million

(8) The following mathematical formula is used to model the population of a small metropolitan population:

$$P_n = P_0 (1+r)^n$$

Where r is the rate of growth per time period

n is the number of time periods

P_0 is the Population when N=0

P_n is the Population when at time period n.

If the population was 100,000 in the year 2000 and the growth rate is 5% every five years, what does the model suggest the population would be in 2010?

A. 110,000

B. 110,250

C. 110,462

D. 162,890

E. 360,000

(9) In 1992 the consumer price index for all urban consumers was 140, using the 1982-1984 baseline, and in 2006 it was 200.

If the average urban household income in 1992 was $39,000, what would the 2006 average urban household income have to be to just maintain the same purchasing power?

A. $27,300

B. $33,456

C. $39,000

D. $55,714

E. $63,250

(10) All three sides of a certain triangle are two inches long. Its area, in square inches, is

A. $\sqrt{3}$

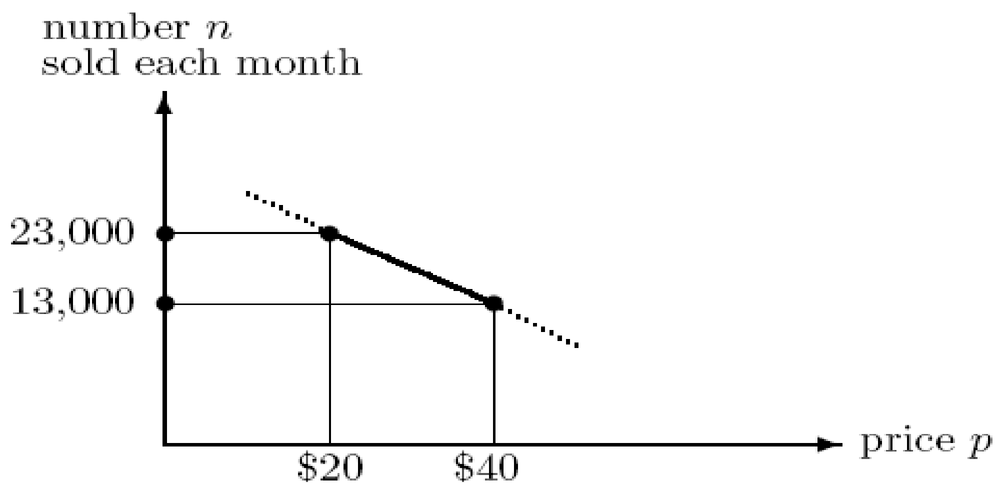
B. $3\frac{\sqrt{3}}{4}$

C. $\sqrt{6}$

D. $3\sqrt{2}$

E. $4\frac{\sqrt{2}}{3}$

(11) The Ace Corporation believes that the retail price *p* of its widgets should be set between 20 and 40 dollars. Marketing research shows that the number of units sold each month, call it *n*, decreases linearly as the price increases, as shown in the graph below:



At what value should the retail price be set to maximize the total revenue per month from the sale of widgets?

A. $24

B. $30

C. $33

D. $35

E. $39

(12) A box contains fuses rated at 10 amperes (A) and 15 A

	10A	15A	Total
Box 1	7	3	10

Suppose fuses are selected at random from the box (one by one without replacement) until a 15A fuse is selected.

What is the probability that a total of two fuses are selected from the box?

A. 7/30

B. 3/7

C. 3/10

D. 1/3

E. None of the above

(13) During a recent trip to the grocery store, you notice that the price of your milk, toothpaste, and cookies totals up to the same amount as the price of the milk and cheese together. You also notice that the cost of milk and cookies is the same as the cost of toothpaste and cheese. Which object can be obtained for the price of two toothpastes?

A. Toothpaste

B. Milk

C. Cheese

D. Cookies

E. None of the above

(14) If the correlation between body weight and annual income were high and positive, we could conclude that:

A. High incomes cause people to eat more food.

B. Low incomes cause people to eat less food.

C. High income people tend to spend a greater proportion of their income on food than low income people, on average.

D. High income people tend to be heavier than low income people, on average.

E. High incomes cause people to gain weight.

(15) A survey on reading ability, conducted in person by trained interviewers on a random sample of households, asked adults to report "How many books have you read in your lifetime?" The results of the survey are questionable because:

A. People cannot reliably recall all the books they've read

B. There is no relation between a person's reading ability and how much they choose to read

C. Some people may exaggerate the number of books because they want to appear to be well-read

D. All of the above

E. Only (A) and (C) are correct

(16) Social, behavioral, and medical research frequently relies on results obtained from a sample of subjects within a population. When it comes to drawing a scientific sample, which of the following is CORRECT?

A. We do not need to randomize if our sample size is sufficiently large.

B. A large sample size always ensures that our sample is representative of the population.

C. If all other things are equal, we need a larger sample size for a larger population.

D. In a properly chosen sample, an estimate will be less variable with a large sample size and hence more precise.

E. In random samples, the randomization ensures that we get precise and accurate estimates.

(17) For one month, 500 elementary students kept a daily record of the hours they spent watching television. The average number of hours per week spent watching television was 28. The researchers conducting the study also obtained report cards for each of the students. They found that the students who did well in school spent less time watching television than those students who did poorly. What statement below describes the best interpretation of these results?[7]

A. The sample of 500 is too small to permit drawing conclusions.

B. If a student decreased the amount of time spent watching television, his or her performance in school would improve.

C. Even though students who did well watched less television, this doesn't necessarily mean watching television hurts school performance.

D. One month is not a long enough period of time to estimate how many hours the students really spend watching television.

E. Since viewing of educational television programs was not measured, the results cannot be interpreted.

(18) A study was conducted to investigate the effectiveness of a new drug for treating Stage 4 AIDS patients. A group of AIDS patients was randomly divided into two groups. One group received the new drug; the other group received a placebo. The difference in mean subsequent survival (those with drugs - those without drugs– was found to be 1.04 years and a 95% confidence interval was found to be 1.04 ± 2.37 years. Based upon this information:

A. We can conclude that the drug was effective because those taking the drug lived, on average, 1.04 years longer.

B. We can conclude that the drug was ineffective because those taking the drug lived, on average, 1.04 years less.

C. We can conclude that the evidence that the drug was effective could be due to chance because the 95% confidence interval covers zero.

D. We can conclude that there is evidence the drug was effective because the 95% confidence interval does not cover zero.

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[7] Question 17 was adapted from the Statistical Reasoning Assessment: Garfield, J.B. (2003). Assessing statistical reasoning. *Statistics Education Research Journal, 2,* 22-38.

E. We can make no conclusions because we do not know the sample size nor the actual mean survival of each group.

(19) A new medication is being tested to determine its effectiveness in the treatment of eczema, an inflammatory condition of the skin. Three hundred patients with eczema were selected to participate in the study. The patients were randomly divided into two groups. Two hundred patients in an experimental group received the medication, while one hundred patients in a control group received no medication. The results after two months are shown below.[8]

	Experimental group (medication)	Control group (no medication)
Improved	80	20
No improvement	120	80

Based on the data the best conclusion about the effectiveness or ineffectiveness of the medication is:

A. Somewhat effective – 80 people improved in the experimental group while only 20 people improved in the control group.

B. Somewhat ineffective – in the control group, 20 people improved even without the medication.

C. Somewhat effective – 40% of the patients in the experimental group improved (80/200), while only 20% improved in the control group (20/100).

D. Somewhat ineffective – in the experimental group, more people didn't get better than did get better.

E. Ineffective – overall only 100 people improved but twice that many people showed no improvement.

(20) The Wall Street Journal recently received 394 readers' responses to a survey on the contribution of universities to the economy. The magazine used the survey results to draw conclusions about the general public's perception of universities. The main problem with using these results is:

A. Selection Bias

B. Insufficient attention to the placebo effect

C. No control group

D. Insufficient number of respondents

E. Interviewer Bias

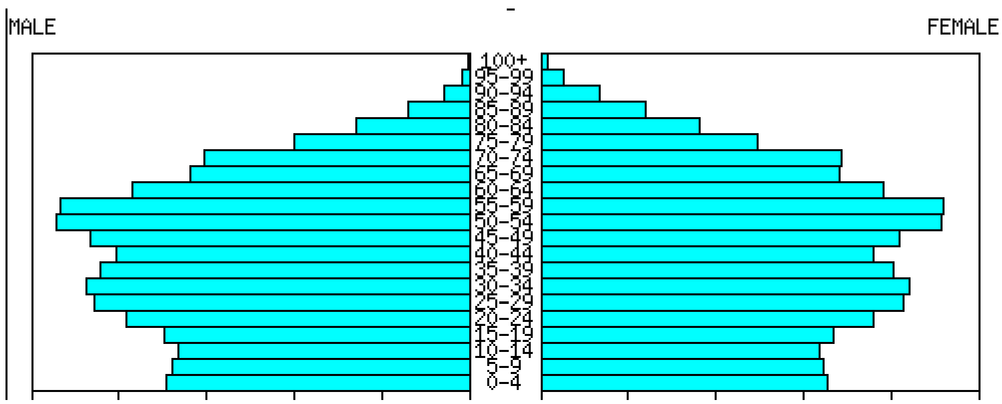
[8] Question 19 was adapted from the Statistical Reasoning Assessment: Garfield, J.B. (2003). Assessing statistical reasoning. *Statistics Education Research Journal, 2,* 22-38.

(21) The local student newspaper is doing an article on the issue of population and the disparity between developed and less-developed nations (less-developed nations tend to have younger populations). To illustrate a point in the article the author has charted the population pyramids for a developed nation and a less-developed nation for 2000, 2010 and 2020. Each Pyramid shows the number of persons within each five-year cohort for Males and Females. With press time fast approaching and the author not answering email, cell phone calls nor text messages, you need to arrange the following charts in the order presented below:

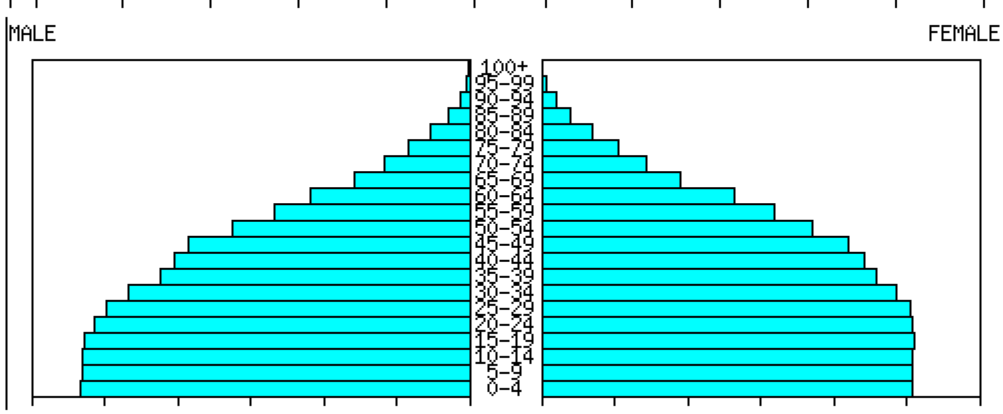
Developed Country, (1) 2000, (2) 2010, (3) 2020
Less Developed Country (4) 2000, (5) 2010, (6) 2020.

Which is the correct sequence?

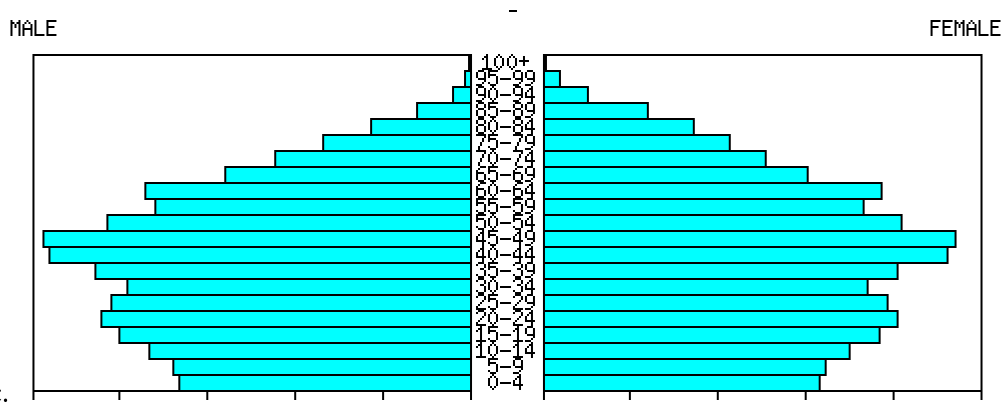
A. d, f, e, b, c, a
B. d, f, b, e, c, a.
C. b, f, d, e, c, a.
D. a, c, e, b, f, d.
E. e, c, a, d, f, b.



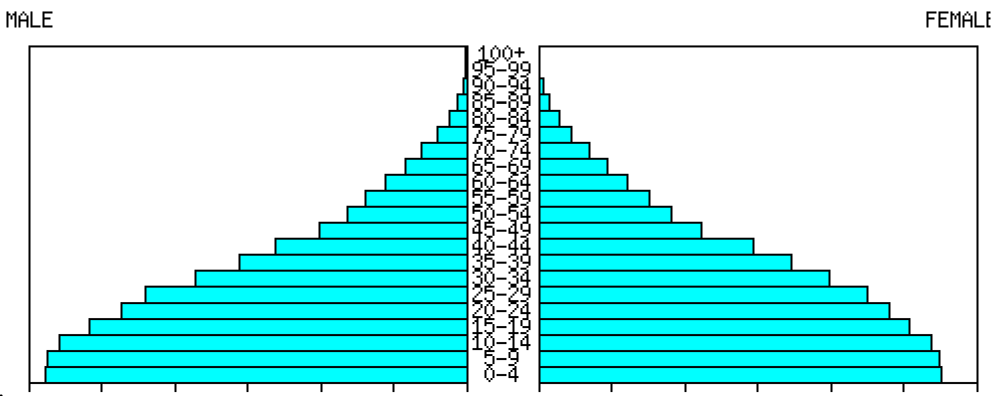
a.



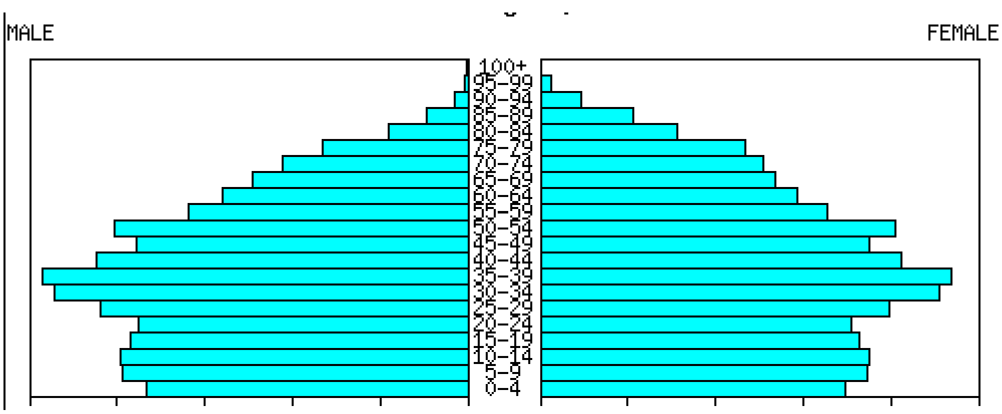
b


MALE
FEMALE
100+
95-99
90-94
85-89
80-84
75-79
70-74
65-69
60-64
55-59
50-54
45-49
40-44
35-39
30-34
25-29
20-24
15-19
10-14
5-9
0-4

c.


MALE
FEMAL
100+
95-99
90-94
85-89
80-84
75-79
70-74
65-69
60-64
55-59
50-54
45-49
40-44
35-39
30-34
25-29
20-24
15-19
10-14
5-9
0-4

d.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10-14
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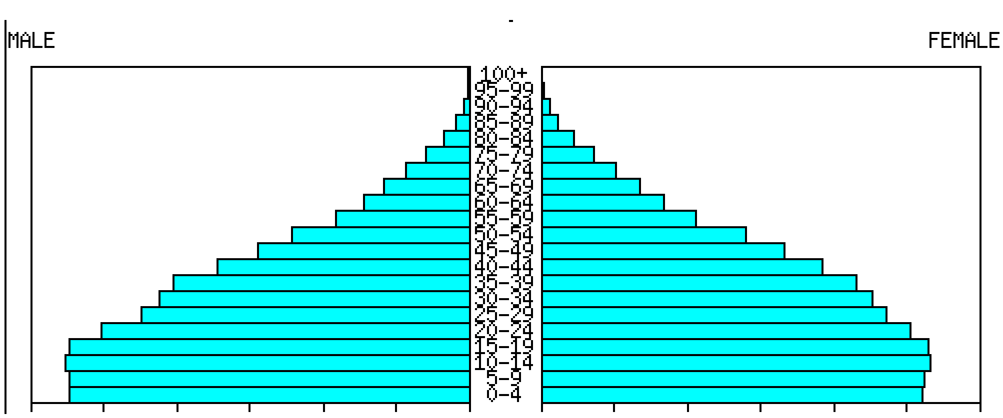
e.


MALE
FEMALE
100+
95-99
90-94
85-89
80-84
75-79
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65-69
60-64
55-59
50-54
45-49
40-44
35-39
30-34
25-29
20-24
15-19
10-14
5-9
0-4

f.

(22) William M. Rohe and Leslie S. Stewart (2006) explored how long a householder would remain in a home in a metropolitan area. They were interested in exploring how tenure (ownership or renting) and various conditions in the neighborhood affected the probability that a householder would remain in the same house for five or more years. After randomly selecting 2,442 neighborhoods (census tracts) among all U.S. metropolitan urban neighborhoods that were not subject to rapid population or housing unit changes, the regression results of the "independent" or "explaining" variables on the "dependent" variable of "Percent of occupied housing units where the householder has resided five or more years" is given below.

Table 1. **Model of Homeownership's Effect on Length of Tenure, 1980 to 1990**

	Regression Coefficient (β)	Statistical Significance (p)	Mean Value
Homeownership[a]			
Percentage of housing units that are owner-occupied[b]	0.3634	0.0001	50.08
Control Variables[a]			
Percentage of householders ages 30 to 61	0.7547	0.0001	34.50
Percentage of households with children	0.1407	0.0001	42.00
Percentage of householders who are black	0.0944	0.0001	24.18
Median household income in tract[c]	-0.000665	0.0001	$14, 563
Mean number of bedrooms per unit	3.795	0.0001	2.28
Percentage of housing units that are vacant	-0.4774	0.0001	6.15
Percentage of owner-occupied units that are condominiums	0.1437	0.0001	9.98
Percentage of occupied housing units built prior to 1940	-0.0147	0.3073	9.80
Percentage of housing units without complete plumbing	0.2296	0.0333	1.62
Intercept	2.070	0.2509	

Note: The dependent variable is the percentage of 1990 occupied housing units where the householder has resided for five or more years. Analysis of variance; Mean of dependent variable = 54.52 percent; N = 2,442; $F = 304.1$; probability > $F = 0.0001$; $R^2 = 0.5557$; adjusted $R^2 = 0.5538$.

[a] 1980 data unless otherwise specified.

[b] Includes units that are vacant and for sale.

[c] 1979 data. Income data reported in each decennial census are annual figures for the previous full year.

Based on the information presented in the table, which of the following statements cannot be supported by the regression results?

A. Home ownership positively affects the duration of tenure.
B. The higher the percentage of householders in a neighborhood who are in the age where they are likely to be raising a family (ages 30 to 61) the more likely the householder would live in a housing unit for five or more years.
C. The higher the percent of vacant housing units in a neighborhood, the more likely householders are to move.
D. The lower the percent of older housing (those built before 1940) in a neighborhood, the more likely a householder would live in a housing unit for five or more years.
E. The larger the housing units in the neighborhood (Mean number of bedrooms per unit) the more likely a householder would live in a housing unit for five or more years.

(23) In the United States, the poor, the less educated, the young and the mobile traditionally vote at much lower rates than the rich, the educated, the middle-aged, and the residentially stable. Stephen Knack and James White (2000) examined how changes in voter registration rules influence these inequalities in turnout. In particular, they analyze whether a wave of states adopting election-day registration prior to the 1994 elections saw an increase in turnout equality based on income, education, age and mobility in 1994 compared to 1990 levels. Using data from the Current Population Study (collected by the U.S. Census and the U.S. Bureau of Labor Statistics), the researchers calculate the ratio of turnout between the relevant groups (presented as a percent – if the ratio is 100%, turnout is equal for the two groups), and regress these values (the dependent variables) on a measure indicating whether or not the state had initiated election-day registration (the independent variable), along with the same ratio in 1990 and overall level of Republican voting in the state (the control variable).

Table 5. Effect of Election-Day Registration on Turnout Ratios

	Income Ratio (Turnout rate for respondents earning less than \$15,000 ÷ Turnout rate for respondents earning more than \$30,000)	**Education Ratio** (Turnout rate for respondents without HS diploma ÷ Turnout rate for respondents with HS diploma)	**Age Ratio** (Turnout rate for respondents under 30 ÷ Turnout rate for respondents over 30)	**Mobility Ratio** (Turnout rate for respondents at current address for <1 year ÷ Turnout rate for respondents at current address for >1 year)
Intercept	28.68 [5.35, 52.01]	34.46 [11.68, 57.24]	51.71 [25.31, 78.11]	29.22 [13.61, 44.83]
1990 Ratio	**-0.41** [-0.65, -0.17]	**-0.38** [-0.61, -0.16]	**-0.72** [-1.10, -0.34]	**-0.63** [-0.85, -0.41]
Election-Day Registration	**6.19** [-6.42, 18.80]	**7.50** [-0.04, 15.04]	**13.72** [6.62, 20.82]	**8.73** [0.73, 16.73]
Republican ID	**0.01** [-0.31, 0.33]	**-0.31** [-0.65, 0.03]	**-0.37** [-0.69, -0.05]	**-0.02** [-0.30, 0.26]

Note: 95% confidence intervals are reported in brackets. The dependent variables are the ratios of turnout for the traditionally less participatory groups to the higher participatory groups (scaled as percents). Higher values represent greater equality of turnout between the groups.

Suppose you are a policy advisor to a state considering adoption of this election reform. Based on the results presented in Table 5, which of the following is the most reasonable conclusion?

A. Election-day registration is not significantly related to turnout in the state.
B. Election-day registration is significantly related to turnout in the state.
C. Election-day registration is significantly related to turnout of currently under-represented groups – the poor, the less educated, the young, and the mobile.
D. Election-day registration is significantly related to turnout of the young, relative to older voters, and among the mobile, relative to residentially stable citizens.
E. Election-day registration is significantly related to turnout among Democrats, relative to Republicans.

(24) Suppose x and y are positive numbers. Then $\frac{1}{x} + \frac{1}{y}$ is equal to:

A. $\frac{x+y}{xy}$

B. $\frac{1}{x+y}$

C. $\frac{xy}{x+y}$

D. $\frac{xy}{x-y}$

E. None of the above

(25) Joe has a rectangular garden, twice as wide as it is long, that is fenced on all four sides and has an area of 1800 square feet. He decides to take the same fencing and use it to enclose a circular dog pen. Which of the following is closest to the area, in square feet, inside the new dog pen?

A. 1920

B. 2000

C. 2300

D. 2600

E. 2900

(26) Consider two functions; *f* and *g*, as defined by the formulas below:

If $f(x) = 2x + 5$ and $g(x) = x^2$ then the composite function $f(g(x)) =$

A. $(2x)^2 + 5$

B. $2x^2 + 5$

C. $(2x + 5)^2$

D. $x^2(2x + 5)$

E. None of the above

(27) If the domain of the function $y = x^3 - 6x^2 + 9x$ is restricted to be the closed interval [0,3], what is the range?

A. {0}

B. [1, 4]

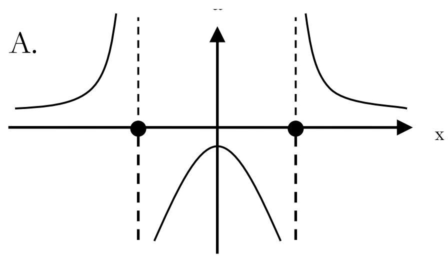
C. [0, 4]

D. (0, 4)

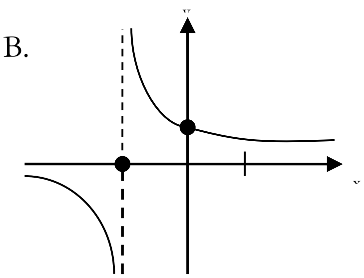
E. (1, 4)

(28) What could be the graph of $y = \dfrac{x-1}{x^2+1}$?

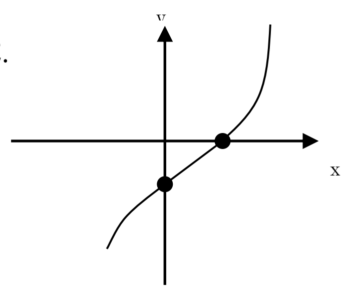
A.



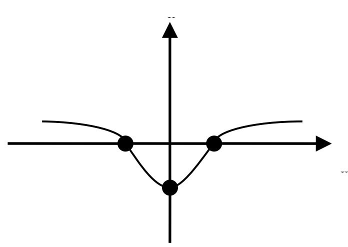
B.



C.



D.



E. None of the above

(29) The radius of a circle is given as 10 cm, with a possible error of measurement equal to 1mm. Use the concept of differentials or other means to estimate the maximum error in the area, in cm^2.

A. 10π

B. 2π

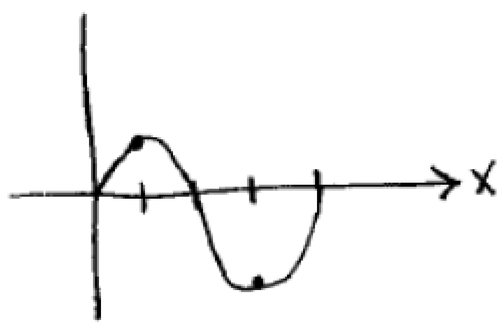
C. 3π

D. π

E. 8π

(30) Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown.

For what values of x is $g'(x) = 0$?



A. 3

B. 1

C. 1, 3

D. 0, 2, 4

E. Not able to determine from the information

Appendix B
Learning Outcomes Matrix

	Learning Outcomes									
Questions	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1. What is the prob. that person lives in City? EASY	X									
2. What is the prob. that person alone is not 65+? MODERATE	X									
3. A store marks up the wholesale price… EASY			X		X					
4. A business meeting includes 6 vice presidents.. EASY			X		X					
5. There are 300 people waiting in line… EASY			X		X					
6. If the average of three numbers x, y, and z… EASY			X			X				
7. The number of websites on the Internet… EASY			X			X				
8. If the population was 100,000…what does the model suggest… MODERATE	X		X							
9. In 1992 the consumer price index…EASY			X							
10. All three sides of a certain triangle are two inches long… MODERATE			X							

11. The Ace Corporation believes that the retail price of its widgets…HARD	X		X	X		X				
12. A box contains fuses…prob. that a total of 2 fuses are selected… MODERATE			X							
13. During a recent trip to the grocery store… MODERATE			X		X			X		
14. If the correlation between body weight… EASY							X			
15. A survey on reading ability… MODERATE										X
16. …research relies on results from a sample… MODERATE										X
17. For 1 month, 500 elementary students… MODERATE							X		X	X
18. A study was conducted…for treating AIDS patients… HARD							X		X	
19. A new medication is being tested… EASY									X	
20. A recent survey by a large American magazine.. MODERATE										X
21. The local student newspaper…developed and less-developed nations MODERATE	X							X		
22. … cannot be supported by the regression results? HARD	X					X	X		X	
23. Suppose you are a policy advisor to the state… HARD	X									

24. Suppose x and y are positive numbers… EASY	X		X							
25. Joe has a rectangular garden… MODERATE			X	X	X			X		
26. Consider two functions, f and g… EASY			X							
27. If the domain of the function y=… MODERATE			X							
28. What could be the graph of… HARD			X							
29. The radius of a circle is given as 10 cm…use the concept of differentials… HARD				X						
30. …where f is the function whose graph is shown… HARD			X	X						

Appendix C
Question Topic Matrix

Question	Calculus/Trig	Stats/Stats Reasoning	General math/Math reasoning
1. What is the prob. that person lives in City? EASY			X
2. What is the prob. that person alone is not 65+? MODERATE			X
3. A store marks up the wholesale price… EASY			X
4. A business meeting includes 6 vice presidents.. EASY			X
5. There are 300 people waiting in line… EASY			X
6. If the average of three numbers x, y, and z… EASY			X
7. The number of websites on the Internet… EASY			X
8. If the population was 100,000…what does the model suggest… MODERATE			X
9. In 1992 the consumer price index… EASY			X
10. All three sides of a certain triangle are two inches long… MODERATE			X
11. The Ace Corporation believes that the retail price of its widgets… HARD			X
12. A box contains fuses…prob. that a total of 2 fuses are selected… MODERATE			X
13. During a recent trip to the grocery store… MODERATE			X
14. If the correlation between body weight… EASY		X	
15. A survey on reading ability… MODERATE		X	
16. …research relies on results from a sample… MODERATE		X	
17. For 1 month, 500 elementary students… MODERATE		X	

	Calculus/Trig	Stats/Stats Reasoning	General math/Math reasoning
18. A study was conducted…for treating AIDS patients… HARD		X	
19. A new medication is being tested… EASY		X	
20. A recent survey by a large American magazine.. MODERATE		X	
21. The local student newspaper…developed and less-developed nations MODERATE			X
22. … cannot be supported by the regression results? HARD		X	
23. Suppose you are a policy advisor to the state… HARD		X	
24. Suppose x and y are positive numbers… EASY			X
25. Joe has a rectangular garden… MODERATE			X
26. Consider two functions, f and g… EASY	X		
27. If the domain of the function y=… MODERATE	X		
28. What could be the graph of… HARD	X		
29. The radius of a circle is given as 10 cm…use the concept of differentials… HARD	X		
30. …where f is the function whose graph is shown… HARD	X		